UE Fluids Problem F13 Solution Fall '07

(10)

1) $\dot{m} = \rho V A = \rho_c V_c A_c$

(1/2) Measured / known quantities: p, T, p_c, also A, A_c

Use additional relations:

(1/2) 2) $p = \rho R T \rightarrow \boxed{\rho = \frac{p}{RT}} \simeq \rho_c$ (low speed flow)

(1/2) 3) $p_o = p_{oc}$ (Bernoulli)

or $p + \frac{1}{2}\rho V^2 = p_c + \frac{1}{2}\rho V_c^2$

From 1) we have $V_c = V \cdot \frac{\rho A}{\rho_c A_c} \simeq V \frac{A}{A_c}$

so $\frac{1}{2}\rho V^2 - \frac{1}{2}\rho \left(V \frac{A}{A_c}\right)^2 = p_c - p$

or $\frac{1}{2}\rho V^2 \left[1 - \left(\frac{A}{A_c}\right)^2\right] = p_c - p$

(4) or $V = \left[\frac{2(p_c - p)}{\rho\left[1 - (A/A_c)^2\right]}\right]^{1/2}$

Finally, $\boxed{\dot{m} = \rho \left[\frac{2(p_c - p)}{\rho\left[1 - (A/A_c)^2\right]}\right]^{1/2} A}$

or equivalent $\boxed{\dot{m} = \rho \left[\frac{2(p_c - p)}{\rho\left[1 + (A_c/A)^2\right]}\right]^{1/2} A_c}$

Or if we plug in explicit ρ expression from above:

$$\dot{m} = \frac{p}{RT} \left[\frac{2(p_c - p)}{\rho\left[1 - (A/A_c)^2\right]}\right]^{1/2} A$$

or $$\dot{m} = \frac{p}{RT} \left[\frac{2(p_c - p)}{\rho\left[1 + (A_c/A)^2\right]}\right]^{1/2} A_c$$